# A Comparative Study to Select the Best Model for Realistic Housing Unit Price Determination*

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## Abstract

The aim of this paper is to compare the following developed models: Multiple Linear Regression Model (MLRM), Principal Components Regression Model (PCRM) and Time Series Analysis Model (TSAM) that could be used to determine realistic Housing Unit Price (HUP) for one-bedroom and two-bedroom housing units. The motive is to select the best model for HUP determination. The MLRM and PCRM were developed using yearly monetary costs over 15 years of Housing Unit Major Components (HUMC) that is, cement, iron rods, aluzinc roofing sheets, coral paint, wood and sand. Multicollinearity analysis was performed to show inputs that are redundant and hence can be removed from the MLRM and PCRM development without necessarily having an effect on the modelling accuracy. With regards to TSAM, observed yearly housing unit prices over 15 years were used to determine HUP for one-bedroom and two-bedroom housing units. To overcome the nonstationarity issues, the yearly nominal housing unit prices were transformed to half-yearly real housing unit prices. The developed models were validated by using them to estimate the known HUP in the 15.5 year. From the results, the percentage absolute deviations of the estimated HUP from the known HUP for the MLRM are 1.27% and 2.02 % for one-bedroom and two-bedroom housing units respectively; for PCRM, they are 1.43% and 0.00% for one-bedroom and two-bedroom housing units respectively; and for TSAM they are all 0.00% for both one-bedroom and two- bedroom housing units. It is thus concluded that TSAM is the best model to be used to determine HUP for both one-bedroom and two-bedroom housing units respectively.

**Keywords:** Comparative Study, Best Model Selection, Housing Unit Price

## 1 Introduction

The issue of obtaining an optimal Housing Unit Price (HUP) acceptable by both sellers and prospective buyers is a great concern in Ghana, Africa and the world at large due to its influence in driving a country's economy. Consequently, both sellers and buyers have to negotiate a lot to find a price that is acceptable to both parties (Merlo and Ortalo-Magne´, 2004). To find an optimal price, researchers have tried some approaches such as Sales Comparison Method (SCM) and Multiple Linear Regression Method (MLRM) to determine the HUP. The SCM suggests that the value of the subject property equals sale prices of similar properties that have been sold recently and are in close proximity to the subject property with due consideration to adjustments for dissimilar characteristics (Isakson, 2002; Kahr and Thomsett, 2005). Since each housing unit is unique in terms of its construction, condition, financing and location (Cupal, 2017), the adjustments made by valuers may be inconsistent and speculative and thus cannot be relied upon to give realistic HUP (Brueggeman and Fisher, 2001). The MLRM can give better estimates of HUP (Chaphalkar and Dhatunde, 2015) but the possible multicollinearity issues in the independent variables and the assumption that they are normally distributed are sometimes not properly resolved by researchers. Again, researchers that have developed MLRMs to estimate HUP have often included intangible housing characteristics as the independent variables, such as quality of neighbourhood (King, 1976) and location (Ayan and Erkin, 2014), all of which may not help in estimating the realistic HUP. To overcome these weaknesses, Boye *et al.* (2017) used MLRM to determine the HUP for one-bedroom and two-bedroom housing units using monetary cost of selected Housing Unit Major Components (HUMC). Boye *et al.* (2018) used Principal Components Regression Model (PCRM) to determine the HUP for one-bedroom and two-bedroom housing units using the same monetary cost of the selected HUMC. Boye *et al.* (2019) used Time Series Analysis Model (TSAM) to determine HUP for one-bedroom and two-bedroom housing units using 15 years real monetary housing unit prices.

This study compares the results obtained from using MLRM, PCRM and TSAM by Boye *et al.* (2017, 2018, 2019) to determine HUP and thus select the best model to determine realistic HUP.

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## 2 Resources and Methods Used

### 2.1 Resources

In developing the MLRM and PCRM, three main resources were used:

(i) Data comprising the quantities of HUMC and HUPs obtained from Regimanuel Gray Estates Ltd., an estate development agency in Accra Metropolitan Area. Table 1 shows the specific HUMC, their units of measurement and the respective quantities needed to construct 1–bedroom and 2– bedroom housing units.

(ii) The respective unit costs of the HUMC over a 15-year period obtained through market survey (Table 2).

(iii) "R" Statistical software.

In formulating the MLRM and PCRM, the quantities of HUMC data in Table 1 was converted to monetary values in US Dollars by using their respective unit costs of the total quantity of each HUMC in Table 2 to obtain primary data for one-bedroom and two-bedroom housing units. See Tables 3 and 4.

**Table 1 Quantities of Housing Unit Major Components**

| Material | Unit | 1 Bedroom | 2 Bedroom |
|---|---|---|---|
| **Cement** | kg | 35 640 | 40 200 |
| **Sand** | $m^3$ | 86 | 99 |
| **16mm Iron Rods** | t | 2 | 3 |
| **Aluzinc Roofing** | $m^2$ | 365 | 678 |
| **Coral Paint** | l | 287.50 | 322 |
| **Wood** | $m^3$ | 4.81 | 6.36 |

The TSAM was developed based on the same HUPs (nominal values) obtained from Regimanuel Gray Estates Ltd. (Tables 4 and 5). For stationarity to be attained, the nominal values were transformed to real monetary HUPs (Table 6) using the relation:

$$RV_i = NV_i \times ER_i \times \left( \frac{CPI_{2003}}{CPI_{NV_i}} \right)$$

for i = 2003 – 2017,

where

$RV_i$ is the i th Real Value;

$NV_i$ is the i th Nominal Value;

$ER_i$ is the i th Exchange Rate;

$CPI_{2003}$ is the Consumer Price Index referenced at 2003; and $CPI_{NVi}$ is the Consumer Price Index for the i th Nominal Value.

**Table 2 Unit Price of Housing Unit Major Components (US $), 2003 – 2017.5**

| | Year | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **Material** | **2003** | **2003.5** | **2004** | **2004.50** | **2005** | **2005.50** | **2006** | **2006.5** | **2007** | **2007.50** |
| **Cement ( kg )** | 0.0017 | 0.0025 | 0.003 | 0.0083 | 0.0147 | 0.017 | 0.0183 | 0.02 | 0.0222 | 0.0243 |
| **Sand ( $m^3$ )** | 17.06 | 22.17 | 27.28 | 32.42 | 37.50 | 40.60 | 43.75 | 58.83 | 73.75 | 84.30 |
| **16mm Iron Rods ( t )** | 159.76 | 167.80 | 175.88 | 183.96 | 192 | 204.60 | 217.20 | 218.89 | 220.55 | 228.60 |
| **Aluzinc Roofing ( $m^2$)** | 3.47 | 3.84 | 4.21 | 4.65 | 4.95 | 5.30 | 5.79 | 6.26 | 6.63 | 7.00 |
| **Coral Paint ( l )** | 0.02 | 0.20 | 0.38 | 0.56 | 0.74 | 0.93 | 1.13 | 1.33 | 1.52 | 1.71 |
| **Wood ( $m^3$ )** | 58.23 | 67.78 | 76.76 | 85.71 | 95.20 | 105.34 | 115.27 | 125.31 | 135.61 | 145.92 |
| **Material** | **2008** | **2008.5** | **2009** | **2009.5** | **2010** | **2010.5** | **2011** | **2011.5** | **2012** | **2012.5** |
| **Cement ( kg )** | 0.0258 | 0.0267 | 0.0438 | 0.0513 | 0.0567 | 0.0617 | 0.0697 | 0.0735 | 0.0742 | 0.0750 |
| **Sand ( $m^3$ )** | 95.00 | 106.14 | 117.19 | 118.40 | 119.69 | 120.50 | 121.25 | 58.83 | 123.00 | 0.0750 |
| 16mm Iron Rods ( t ) | 236.73 | 245.98 | 255.15 | 266.60 | 278.05 | 282.78 | 287.50 | 295.00 | 302.50 | 311.25 |
| **Aluzinc Roofing ( $m^2$)** | 7.47 | 7.92 | 8.31 | 8.70 | 9.15 | 9.58 | 9.99 | 10.40 | 10.83 | 11.34 |
| **Coral Paint ( l )** | 1.91 | 2.11 | 2.30 | 2.50 | 2.69 | 2.89 | 3.08 | 3.27 | 3.47 | 3.68 |
| **Wood ( $m^3$ )** | 155.95 | 165.99 | 176.30 | 186.60 | 196.64 | 206.67 | 216.98 | 227.29 | 237.32 | 247.36 |
| **Material** | **2013** | **2013.5** | **2014** | **2014.5** | **2015** | **2015.5** | **2016** | **2016.5** | **2017** | **2017.5** |
| **Cement ( kg )** | 0.0780 | 0.1090 | 0.1375 | 0.1383 | 0.1417 | 0.1488 | 0.1533 | 0.1583 | 0.1658 | 0.1717 |
| **Sand ( $m^3$ )** | 125.00 | 131.20 | 137.5 | 143.76 | 150.00 | 155.10 | 160.22 | 165.37 | 170.40 | 178.00 |
| 16mm Iron Rods ( t ) | 320.00 | 328.70 | 337.50 | 353.43 | 369.25 | 377.30 | 385.37 | 393.44 | 401.49 | 409.55 |
| **Aluzinc Roofing ( $m^2$)** | 11.67 | 12.00 | 12.51 | 12.82 | 13.13 | 13.50 | 13.87 | 14.25 | 14.60 | 14.97 |
| **Coral Paint ( l )** | 3.86 | 4.05 | 4.25 | 4.46 | 4.65 | 4.83 | 5.01 | 5.19 | 5.37 | 5.54 |
| **Wood ( $m^3$ )** | 257.66 | 267.97 | 278.00 | 288.04 | 298.35 | 307.81 | 316.79 | 325.90 | 335.23 | 345.16 |

**Table 3 Price of Housing Unit and Cost of Total Quantity of HUMC (US $) 2003 – 2017**
(Primary Data for One-Bedroom)

| Year | Housing Unit Price (HUP) | Cost of Cement (CC) | Cost of Sand (CS) | Cost of Iron Rods (CIR) | Cost of Roofing (CR) | Cost of Paint (CP) | Cost of Wood (WP) |
|---|---|---|---|---|---|---|---|
| **2003** | 31 455.00 | 60.59 | 1 467.16 | 319.52 | 1 266.55 | 5.75 | 280.33 |
| **2003.5** | 33 260.00 | 89.10 | 1 906.62 | 335.60 | 1 401.60 | 57.50 | 326.30 |
| **2004** | 35 065.00 | 106.92 | 2 346.08 | 351.76 | 1 536.65 | 109.25 | 369.53 |
| **2004.5** | 36 870.00 | 295.81 | 2 788.12 | 367.92 | 1 697.25 | 161.00 | 412.62 |
| **2005** | 38 675.00 | 523.91 | 3 225.00 | 384.16 | 1 806.75 | 212.75 | 458.30 |
| **2005.5** | 40 587.50 | 605.88 | 3 491.60 | 409.20 | 1 934.50 | 267.38 | 507.12 |
| **2006** | 42 500.00 | 652.21 | 3 762.50 | 434.40 | 2 113.35 | 324.88 | 554.92 |
| **2006.5** | 42 500.00 | 712.80 | 5 059.38 | 439.78 | 2 284.90 | 382.38 | 603.26 |
| **2007** | 42 500.00 | 791.21 | 6 342.50 | 441.10 | 2 419.95 | 437.00 | 652.84 |
| **2007.5** | 44 604.00 | 866.05 | 7 249.80 | 457.20 | 2 555.00 | 491.63 | 702.47 |
| **2008** | 46 708.00 | 919.51 | 8 170.00 | 473.46 | 2 726.55 | 549.13 | 750.76 |
| **2008.5** | 49 020.00 | 951.56 | 9 128.04 | 491.96 | 2 890.80 | 606.63 | 799.09 |
| **2009** | 51 332.00 | 1 561.03 | 10 078.34 | 510.30 | 3 033.15 | 661.25 | 848.73 |
| **2009.5** | 51 332.00 | 1 828.33 | 10 182.40 | 533.20 | 3 175.50 | 718.75 | 898.31 |
| **2010** | 51 332.00 | 2 020.79 | 10 293.34 | 556.10 | 3 339.75 | 773.38 | 946.65 |
| **2010.5** | 53 873.00 | 2 198.99 | 10 363.00 | 565.56 | 3 496.70 | 830.88 | 994.93 |
| **2011** | 56 414.00 | 2 484.11 | 10 427.50 | 575.00 | 3 646.35 | 885.50 | 1 044.56 |
| **2011.5** | 59 206.50 | 2 619.54 | 10 500.60 | 590.00 | 3 796.00 | 940.13 | 1 094.20 |
| **2012** | 61 999.00 | 2 644.49 | 10 578.00 | 605.00 | 3 952.95 | 997.63 | 1 142.48 |
| **2012.5** | 61 999.00 | 2 673.00 | 10 668.30 | 622.50 | 4 139.10 | 1 058.00 | 1 190.82 |
| **2013** | 61 999.00 | 2 779.92 | 10 750.00 | 640.00 | 4 259.55 | 1 109.75 | 1 240.40 |
| **2013.5** | 65 068.00 | 3 884.76 | 11 283.20 | 657.40 | 4 380.00 | 1 164.38 | 1 290.03 |
| **2014** | 68 137.00 | 4 900.50 | 11 825.00 | 675.00 | 4 566.15 | 1 221.88 | 1 338.32 |
| **2014.5** | 69 942.00 | 4 929.01 | 12 363.36 | 706.86 | 4 679.30 | 1 282.25 | 1 386.65 |
| **2015** | 71 747.50 | 5 050.19 | 12 900.00 | 738.50 | 4 792.45 | 1 336.88 | 1 436.29 |
| **2015.5** | 73 552.00 | 5 303.23 | 13 338.60 | 754.60 | 4 927.50 | 1 388.63 | 1 481.83 |
| **2016** | 75 357.50 | 5 463.61 | 13 778.92 | 770.74 | 5 062.55 | 1 440.38 | 1 525.06 |
| **2016.5** | 77 162.00 | 5 641.81 | 14 221.82 | 786.88 | 5 201.25 | 1 492.13 | 1 568.92 |
| **2017** | 78 967.50 | 5 909.11 | 14 654.40 | 802.98 | 5 329.00 | 1 543.88 | 1 613.83 |
| **2017.5** | | 6 119.39 | 15 339.66 | 819.10 | 5 464.05 | 1592.75 | 1 661.63 |

**Table 4 Price of Housing Unit and Cost of Total Quantity of HUMC (US $) 2003 – 2017**

(Primary Data for Two-Bedroom)

| Year | Housing Unit Price (HUP) | Cost of Cement (CC) | Cost of Sand (CS) | Cost of Iron Rods (CIR) | Cost of Roofing (CR) | Cost of Paint (CP) | Cost of Wood (CW) |
|---|---|---|---|---|---|---|---|
| **2003** | 34 500.00 | 68.34 | 1 688.94 | 479.28 | 2 352.66 | 6.44 | 370.49 |
| **2003.5** | 37 280.00 | 100.50 | 2 194.83 | 503.40 | 2 603.52 | 64.40 | 431.26 |
| **2004** | 40 070.00 | 120.60 | 2 700.72 | 527.64 | 2 854.38 | 122.36 | 488.39 |
| **2004.5** | 41 880.00 | 333.66 | 3 209.58 | 551.88 | 3 152.70 | 180.32 | 545.34 |
| **2005** | 43 680.00 | 590.94 | 3 712.50 | 576.00 | 3 356.10 | 238.28 | 605.72 |
| **2005.5** | 45 841.00 | 683.40 | 4 019.40 | 613.80 | 3 593.40 | 299.46 | 670.24 |
| **2006** | 48 000.00 | 735.66 | 4 331.25 | 651.60 | 3 925.62 | 363.86 | 733.42 |
| **2006.5** | 48 000.00 | 804.00 | 5 825.17 | 656.67 | 4 244.28 | 428.26 | 797.30 |
| **2007** | 48 000.00 | 892.44 | 7 301.25 | 661.65 | 4 495.14 | 489.44 | 862.83 |
| **2007.5** | 53 579.00 | 976.86 | 8 345.70 | 685.80 | 4 746.00 | 550.62 | 928.43 |
| **2008** | 59 160.00 | 1 037.16 | 9 405.00 | 710.19 | 5 064.66 | 615.02 | 992.25 |
| **2008.5** | 64 001.00 | 1 073.34 | 10 507.86 | 737.94 | 5 369.76 | 679.42 | 1 056.13 |
| **2009** | 68 840.00 | 1 760.76 | 11 601.81 | 765.45 | 5 634.18 | 740.60 | 1 121.73 |
| **2009.5** | 74 419.00 | 2 062.26 | 11 721.60 | 799.8 | 5 898.60 | 805.00 | 1 187.26 |
| **2010** | 80 000.00 | 2 279.34 | 11 849.31 | 834.15 | 6 203.70 | 866.18 | 1 251.14 |
| **2010.5** | 80 000.00 | 2 480.34 | 11 929.50 | 848.34 | 6 495.24 | 930.58 | 1 314.96 |
| **2011** | 80 000.00 | 2 801.94 | 12 003.75 | 862.50 | 6 773.22 | 991.76 | 1 380.56 |
| **2011.5** | 89 001.00 | 2 954.70 | 12 087.90 | 885.00 | 7 051.20 | 1 052.94 | 1 446.16 |
| **2012** | 98 000.00 | 2 982.84 | 12 177.00 | 907.50 | 7 342.74 | 1 117.34 | 1 509.97 |
| **2012.5** | 103 671.00 | 3 015.00 | 12 280.95 | 933.75 | 7 688.52 | 1 184.96 | 1 573.85 |
| **2013** | 109 340.00 | 3 135.60 | 12 375.00 | 960.00 | 7 912.26 | 1 242.92 | 1 639.39 |
| **2013.5** | 114 919.00 | 4 381.80 | 12 988.80 | 986.10 | 8 136.00 | 1 304.10 | 1 704.99 |
| **2014** | 120 500.00 | 5 527.50 | 13 612.50 | 1 012.50 | 8 481.78 | 1 368.50 | 1 768.80 |
| **2014.5** | 126 727.00 | 5 559.66 | 14 232.24 | 1 060.29 | 8 691.96 | 1 436.12 | 1 832.68 |
| **2015** | 132 955.00 | 5 696.34 | 14 850.00 | 1 107.75 | 8 902.14 | 1 497.30 | 1 898.28 |
| **2015.5** | 139 183.00 | 5 981.76 | 15 354.90 | 1 131.90 | 9 153.00 | 1 555.26 | 1 958.47 |
| **2016** | 145 410.00 | 6 162.66 | 15 861.78 | 1 156.10 | 9 403.86 | 1 613.22 | 2 015.61 |
| **2016.5** | 151 637.00 | 6 363.66 | 16 371.63 | 1 180.32 | 9 661.50 | 1 671.18 | 2 073.57 |
| **2017** | 157 865.00 | 6 665.16 | 16 869.60 | 1 204.47 | 9 898.80 | 1 729.14 | 2 132.93 |
| **2017.5** | | 6 902.34 | 17 424.27 | 1 228.65 | 10 149.66 | 1 783.88 | 2 193.53 |

**Table 4 Half- Yearly Prices of One-Bedroom Housing Unit (US $) 2003 – 2017**

| Year | 2003 | 2003.5 | 2004 | 2004.5 | 2005 | 2005.5 | 2006 | 2006.5 | 2007 | 2007.5 | 2008 | 2008.5 | 2009 | 2009.5 | 2010 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Price ($) | 31455 | 33260 | 35065 | 36870 | 38675 | 40587.5 | 42500 | 42500 | 42500 | 44604 | 46708 | 49020 | 51332 | 51332 | 51332 |

| Year | 2010.5 | 2011 | 2011.5 | 2012 | 2012.5 | 2013 | 2013.5 | 2014 | 2014.5 | 2015 | 2015.5 | 2016 | 2016.5 | 2017 | 2017.5 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Price ($) | 53873 | 56414 | 59206.5 | 61999 | 61999 | 61999 | 65068 | 68137 | 69942 | 71747.5 | 73552 | 75357.5 | 77162 | 78967.5 | 83 600 |

**Table 5 Half- Yearly Prices of Two-Bedroom Housing Unit (US $) 2003 – 2017**

| Year | 2003 | 2003.5 | 2004 | 2004.5 | 2005 | 2005.5 | 2006 | 2006.5 | 2007 | 2007.5 | 2008 | 2008.5 | 2009 | 2009.5 | 2010 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Price ($) | 34500 | 37280 | 40070 | 41880 | 43680 | 45841 | 48000 | 48000 | 48000 | 53579 | 59160 | 64001 | 68840 | 74419 | 80000 |

| Year | 2010.5 | 2011 | 2011.5 | 2012 | 2012.5 | 2013 | 2013.5 | 2014 | 2014.5 | 2015 | 2015.5 | 2016 | 2016.5 | 2017 | 2017.5 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Price ($) | 80000 | 80000 | 89001 | 98000 | 103671 | 109340 | 114919 | 120500 | 126727 | 132955 | 139183 | 145410 | 151637 | 157865 | 169 000 |

**Table 6 Exchange Rate, Consumer Price Index (CPI) and Real Monetary Values of Housing Unit Prices**

| | | | Real Monetary Values | |
|---|---|---|---|---|
| **Date** | **Exchange Rate** | **CPI** | **One-Bedroom** | **Two-Bedroom** |
| **2003.01** | 0.8375 | 28.2400 | 26 343.56 | 28 893.75 |
| **2003.07** | 0.8450 | 34.2300 | 23 186.58 | 25 989.05 |
| **2004.01** | 0.8675 | 36.4200 | 23 586.75 | 26 953.40 |
| **2004.07** | 0.9032 | 39.3600 | 23 892.78 | 27 139.40 |
| **2005.01** | 0.9000 | 42.5400 | 23 106.81 | 26 097.11 |
| **2005.07** | 0.9040 | 46.2000 | 22 427.63 | 25 330.59 |
| **2006.01** | 0.9095 | 47.9600 | 22 760.26 | 25 705.70 |
| **2006.07** | 0.9225 | 52.1700 | 21 222.63 | 23 969.09 |
| **2007.01** | 0.9210 | 53.2000 | 20 777.90 | 23 466.80 |
| **2007.07** | 0.9307 | 57.4600 | 20 402.46 | 24 507.75 |
| **2008.01** | 0.9688 | 60.0100 | 21 294.45 | 26 971.39 |
| **2008.07** | 1.1550 | 67.9900 | 23 516.62 | 30 703.54 |
| **2009.01** | 1.3402 | 71.9100 | 27 016.76 | 36 231.46 |
| **2009.07** | 1.4965 | 81.9000 | 26 487.79 | 38 400.89 |
| **2010.01** | 1.4312 | 82.5500 | 25 132.53 | 39 168.59 |
| **2010.07** | 1.4383 | 89.6400 | 24 410.88 | 36 249.52 |
| **2011.01** | 1.5637 | 90.0600 | 27 661.33 | 39 226.19 |
| **2011.07** | 1.5126 | 97.1800 | 26 024.43 | 39 120.71 |
| **2012.01** | 1.6808 | 97.9000 | 30 059.57 | 47 514.28 |
| **2012.07** | 1.9549 | 106.4500 | 32 153.50 | 53 765.15 |
| **2013.01** | 1.9035 | 106.5000 | 31 293.39 | 55 188.30 |
| **2013.07** | 2.0800 | 113.6000 | 33 644.74 | 59 421.22 |
| **2014.01** | 2.3800 | 121.2000 | 37 785.23 | 66 823.02 |
| **2014.07** | 3.4650 | 131.0000 | 52 243.79 | 94 659.85 |
| **2015.01** | 3.3300 | 141.1000 | 47 817.70 | 88 610.79 |
| **2015.07** | 3.8200 | 154.5000 | 51 356.34 | 97 181.98 |
| **2016.01** | 3.9533 | 168.0000 | 50 077.39 | 96 629.44 |
| **2016.07** | 3.8965 | 180.3000 | 47 092.00 | 92 544.12 |
| **2017.01** | 4.3615 | 190.3400 | 51 099.76 | 102 154.23 |

**Source:** *www.bog.gov.gh*, Accessed: July 2, 2017

## 2.2 Development of Models

According to (Brooks, 2008) and Boye *et al.* (2017), in developing the MLRM, the possible multicollinearity issues in the independent variables and the assumption that they are normally distributed that are sometimes not properly resolved by researchers were taken care of by log transforming the sample data since the matrix scatter plot revealed that the HUMC were strongly correlated. Consequently, $\hat{\beta}_{OLS} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y}$ is the deduced model coefficients whose numerical values were determined from sample data. The developed model is $\hat{\mathbf{Y}}_i = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y}(\mathbf{x}'_i)$ and it was validated by using it to estimate the known HUP in the 15.5th year.

According to Efron and Tibshirani (1994) and Boye *et al.* (2018), in developing the PCRM, multicollinearity which existed among the sample

data and could have caused wrong statistical inferences was resolved by log transformation of the data and Principal Component Analysis (PCA) technique which reduced the dimensionality of the predictive variables and accounted for multicollinearity problems was applied. Scree plots were used to determine the minimum number of principal components which accounted for the total variation in the HUP. Subsequently, PCA method was used to derive the model coefficients $\hat{\beta} = \mathbf{V}\hat{\gamma}$ from the sample data. The three principal components which explained 99.37% of the total variation in the one-bedroom and two-bedroom housing units respectively resulted in the following PCR model $\hat{\mathbf{Y}}_i = \beta_0 + \sum_{j=1}^{p} \hat{\beta}_j \mathbf{x}_i$ and it was validated by using it to estimate the known HUP in the 15.5th year.

According to Montgomery *et al.* (2008) and Boye *et al.* (2019) in developing the TSAM, the nonstationarity among the observed yearly housing unit prices over 15 years which could have caused wrong model coefficients was taken care of by converting the observed yearly housing unit prices over 15 years to yearly real monetary housing unit prices over 15 years. $Y_t = \theta_0 + \sum_{P=1}^{P} \theta_P Y_{t-P} + \sum_{q=1}^{Q} \phi_q \varepsilon_{t-q}$ was the function used to determine the type of Autoregressive Moving Average (ARMA) model the sample data depicts. Consequently, the model parameters were determined using the sample data.

## 3 Results and Discussion

According to Boye *et al.* (2017), ordinary least squares method was used to derive the MLRM coefficients $\hat{\beta}_i$, and the corresponding equations for one-bedroom and two-bedroom housing units are as follows:

$\log_e (HUP_{MLRM})_{1\text{-Bed}} = 1.017 - 2.225 \times 10^{-5} \times CC + 2.512 \times 10^{-6} \times CS + 6.016 \times 10^{-4} \times CIR + 1.985 \times 10^{-4} \times CR + 5.694 \times 10^{-4} \times CP - 7.437 \times 10^{-4} \times CW$

$\log_e (HUP_{MLRM})_{2\text{-Bed}} = 5.760 - 7.501 \times 10^{-7} \times CC + 2.935 \times 10^{-6} \times CS + 1.898 \times 10^{-3} \times CIR + 6.695 \times 10^{-4} \times CR - 9.157 \times 10^{-3} \times CP + 6.136 \times 10^{-3} \times CW$

According to Boye *et al.* (2018), principal components regression method was used to derive the PCRM coefficients $\hat{\beta}_i$, and the corresponding equations for one-bedroom and two-bedroom housing units are as follows:

$\log_e (HUP_{PCRM})_{1\text{-Bed}} = 10.866658 - 0.2 \times 10^{-4} \times CC + 0.1 \times 10^{-5} \times CS - 7.18 \times 10^{-4} \times CIR - 2.37 \times 10^{-4} \times CR + 1.275 \times 10^{-3} \times CP + 2.48 \times 10^{-4} \times CW$

$\log_e (HUP_{PCRM})_{2\text{-Bed}} = 11.231345 + 0.7 \times 10^{-5} \times CC - 0.4 \times 10^{-5} \times CS - 1.182 \times 10^{-3} \times CIR - 1.54 \times 10^{-4} \times CR + 1.633 \times 10^{-3} \times CP + 4.24 \times 10^{-4} \times CW$

According to Boye *et al.* (2019), time series analysis method was used to derive the TSAM coefficients $\theta$ and $\phi$, and the corresponding equations for one-bedroom and two-bedroom housing units are as follows:

$\hat{HUP}_{t1\text{-Bed}} = 440.531 - 0.181\, y_{t-1} + 0.022\, y_{t-2} + 0.993\, e_{t-1}$

and

$\hat{HUP}_{t2\text{-Bed}} = 278.474 - 0.166\, y_{t-1} + 0.035\, y_{t-2} - 0.062\, y_{t-3} + 0.994\, e_{t-1}$

*Model Validation*

In order to find the best of the three developed models for one-bedroom and two-bedroom housing units, they were used to estimate the known HUP in the 15.5 year for one-bedroom and two-bedroom housing units. Table 7 is a summary of the results. From the results, the percentage absolute deviations, ($\Delta$ %), of all the estimates of the HUP from the known HUP are between 0.00% and 2.02%, which are considered to be satisfactory. However, the TSAM gave the best estimate of the HUP for one-bedroom and two-bedroom housing units with both $\Delta\%_{TSAM,1-bed}$ and $\Delta\%_{TSAM,2-bed} = 0.00$, meaning the TSAMs are very good and the best.

**Table 7 Estimated HUP and Respective Percentage Absolute Deviation ($\Delta$ %) from the Known HUP**

| Housing Unit | Known HUP ($) | Estimated HUP ($) from MLRM | $\Delta$ % | Estimated HUP ($) from PCRM | $\Delta$ % | Estimated HUP ($) from TSAM | $\Delta$ % |
|---|---|---|---|---|---|---|---|
| 1-Bedroom | 83 600.00 | 82 530.24 | 1.27 | 82 401.32 | 1.43 | 83 618.82 | 0.00 |
| 2-Bedroom | 169 000.00 | 172 413.10 | 2.02 | 169 022.20 | 0.00 | 169 104.99 | 0.00 |

## 4 Conclusions and Recommendation

Table 7 shows a summary result of the study. The percentage absolute deviation (Δ%) of the estimated HUP for MLRM, PCRM and TSAM for one-bedroom and two-bedroom housing units in the 15.5 year are between 0.00% and 2.02%, which are considered to be satisfactory. However, the TSAM gave the best estimate of the HUP for one-bedroom and two-bedroom housing units with percentage absolute deviation of both being 0.00%.

For future research, it is recommended that the developed approach should be improved upon to determine the HUP of other types of housing units since it would give prospective house owners a timely, good idea of the price of a house they intend to purchase.

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