Time Series Analysis Model for Estimating Housing Unit Price

[1]P. Boye, [1]D. Mireku-Gyimah and [2]H. Sadiq
[1]University of Mines and Technology, Tarkwa, Ghana
[2]University of Cape Town, Department of Statistics, South Africa

Boye, P., Mireku-Gyimah, D. and Sadiq, H. (2019), "Time Series Analysis Model for Estimating the Price of a Housing Unit", *Ghana Journal of Technology*, Vol. 3, No. 2, pp. 35 - 41.

Abstract

This paper uses observed housing unit prices over fifteen (15) years to develop Time Series Analysis Model (TSAM) for determining Housing Unit Price (HUP) for one-bedroom and two-bedroom housing units. In the modeling, the observed prices were converted to real monetary values and AutoCorrelation Functions (ACF) and Partial AutoCorrelation Function (PACF) plots were used to estimate the model coefficients for one-bedroom and two-bedroom housing units respectively. The resultant developed models for one-bedroom and two-bedroom housing units are Autoregressive Integrated Moving Average, ARIMA (2, 1, 1) and ARIMA (3, 1, 1) respectively. The specific model for one-bedroom housing unit is $\hat{HUP}_{t1\text{-}Bed}$ = 440.531- 0.181 y_{t-1} + 0.022 y_{t-2} + 0.993 e_{t-1} and that for two-bedroom hosing unit is $\hat{HUP}_{t2\text{-}Bed}$ = 278.474 - 0.166 y_{t-1} + 0.035 y_{t-2} - 0.062 y_{t-3} + 0.994 e_{t-1}. The TSAM was validated by using it to estimate the known HUP in the 15.5th year. From the results, the percentage absolute deviations of the estimated HUP from the known HUP for one-bedroom and two-bedroom housing units are all equal to 0.00% respectively, meaning that both models are good. The approach presented in this paper is a valuable contribution to the body of knowledge in modeling.

Keywords: Time Series Analysis, Housing Unit Price, Nominal and Real Monetary Housing Unit Prices

1 Introduction

The determination of a realistic housing unit price in Ghana is an issue of great concern owing to the fact that there seems to be a large disparity between what sellers think a house is worth and what buyers are prepared to pay for a housing unit. Consequently, both sellers and buyers have to negotiate a lot to find a price that is acceptable to both parties. Boye *et al.* (2017) developed Multiple Linear Regression Model (MLRM) to determine Housing Unit Price (HUP) for one-bedroom and two-bedroom housing units using selected Housing Unit Major Components (HUMC). Boye *et al.* (2018) also developed Principal Components Regression Model (PCRM) to determine HUP for one-bedroom and two-bedroom housing units using the same selected HUMC used by Boye *et al.* (2017).

This study determines the HUP based on the nominal values of housing unit prices, resolving the issues of non-stationarity by converting the nominal housing unit prices to real monetary housing unit prices.

2 Resources and Methods Used

2.1 Resources

This study used two main resources:

(i) Housing Unit Prices (HUP) over a period of 15 consecutive years, for one-bedroom and two-bedroom housing units obtained from Regimanuel Gray Estates Ltd., an estate development agency in Accra Metropolitan Area. See Table 1.

(ii) "R" statistical software .

2.2.1 Development of ARIMA Models

Box-Jenkins Autoregressive Integrated Moving Average model consists of the Autoregressive (AR (p)) model and the Moving Average (MA (q)) model. When these models are put together, the Autoregressive Moving Average (ARMA (p, q)) model is formed. ARMA processes form the core of time-series analysis. The ARMA class can be decomposed into two smaller classes, autoregressive (AR) processes and moving average (MA) processes.

According to Gebhard *et al.,* (2013) and Enders, (2015) the first order moving average, abbreviated as MA (1), is the simplest non-degenerated time-series process defined as

$$Y_t = \phi_0 + \phi_1 \varepsilon_{t-1} + \varepsilon_t \tag{1}$$

where ϕ_0 and ϕ_1 are unknown model coefficients whose actual values would be determined from sample data; and ε_t is a white noise process.

The first order autoregressive process abbreviated AR (1) has the following dynamics:

**Manuscript received August 08, 2018*
Revised version accepted February 14, 2019

$$Y_t = \theta_0 + \theta_1 Y_{t-1} + \varepsilon_t \qquad (2)$$

where ϕ_0 and ϕ_1 are the unknown model coefficients whose actual values would be determined from sample data; and ε_t is a white noise process.

When the Autoregressive and the Moving Average processes are put together, they yield the complete class of ARMA processes. An Autoregressive Moving Average process with orders P and Q, ARMA (P, Q), has the following dynamics:

$$Y_t = \theta_0 + \sum_{p=1}^{P} \theta_p Y_{t-p} + \sum_{q=1}^{Q} \phi_q \varepsilon_{t-q} \qquad (3)$$

Assumptions

(i) The ε_t are independent identically distributed.

(ii) $\varepsilon_t \sim N\left(0, \sigma^2\right)$

Hypothesis Test

H_0 : Series is not stationary

H_1 : Series is stationary

2.2.2 Analysis of Data

In formulating the ARIMA model, the observed HUPs (nominal values) in Tables 1 and 2 were used to plot ACF graphs for one-bedroom and two-bedroom housing units to assess stationarity process among the data. See Figs. 1 and 2. Since these figures appear not to be stationary, the one-bedroom and two-bedroom data in Tables 1 and 2 were differenced a couple of times and that also did not result in stationarity. Consequently, the nominal values of the housing unit prices in Tables 1 and 2 were converted to real monetary values of housing unit prices shown in Table 4 by using the relation

$$RV_i = NV_i \times ER_i \times \left(\frac{CPI_{2003}}{CPI_{NV_i}} \right)$$

and Table 3 for i = 2003 – 2017,

where RV_i is the i th Real Value;

NV_i is the i th Nominal Value;

ER_i is the i th Exchange Rate;

CPI_{2003} is the consumer Price Index referenced at 2003; and CPI_{NVi} is the Consumer Price Index for the i th Nominal Value.

The real monetary values of the housing unit prices in Table 4 were then used to plot ACF and PACF graphs for one-bedroom and two-bedroom housing units respectively by differencing once to assess the stationary process. See Figs. 3 to 6. Since the resultant ACFs and PACFs for one-bedroom and two-bedroom housing units respectively obtained from the real monetary housing unit prices attained stationarity, consequently, they were used to generate the one-bedroom and two-bedroom housing unit price models coefficients. Formal stationarity test was further carried out to ensure that the real monetary values of the housing unit prices used to plot the ACFs and PACFs graphs were truly stationary. See Tables 5 and 6.

Table 1 Half- Yearly Prices of One-Bedroom Housing Unit (US $) 2003 – 2017

Year	2003	2003.5	2004	2004.5	2005	2005.5	2006	2006.5	2007	2007.5	2008	2008.5	2009	2009.5	2010
Price ($)	31455	33260	35065	36870	38675	40587.5	42500	42500	42500	44604	46708	49020	51332	51332	51332

Year	2010.5	2011	2011.5	2012	2012.5	2013	2013.5	2014	2014.5	2015	2015.5	2016	2016.5	2017	2017.5
Price ($)	53873	56414	59206.5	61999	61999	61999	65068	68137	69942	71747.5	73552	75357.5	77162	78967.5	83 600

Table 2 Half- Yearly Prices of Two-Bedroom Housing Unit (US $) 2003 – 2017

Year	2003	2003.5	2004	2004.5	2005	2005.5	2006	2006.5	2007	2007.5	2008	2008.5	2009	2009.5	2010
Price ($)	34500	37280	40070	41880	43680	45841	48000	48000	48000	53579	59160	64001	68840	74419	80000

Year	2010.5	2011	2011.5	2012	2012.5	2013	2013.5	2014	2014.5	2015	2015.5	2016	2016.5	2017	2017.5
Price ($)	80000	80000	89001	98000	103671	109340	114919	120500	126727	132955	139183	145410	151637	157865	169 000

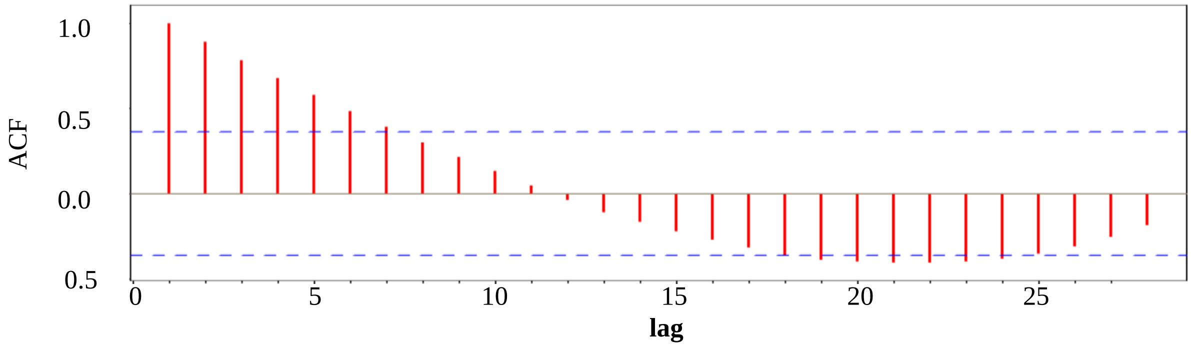

Fig. 1 Non-Stationary ACF Nominal Series for One-Bedroom House

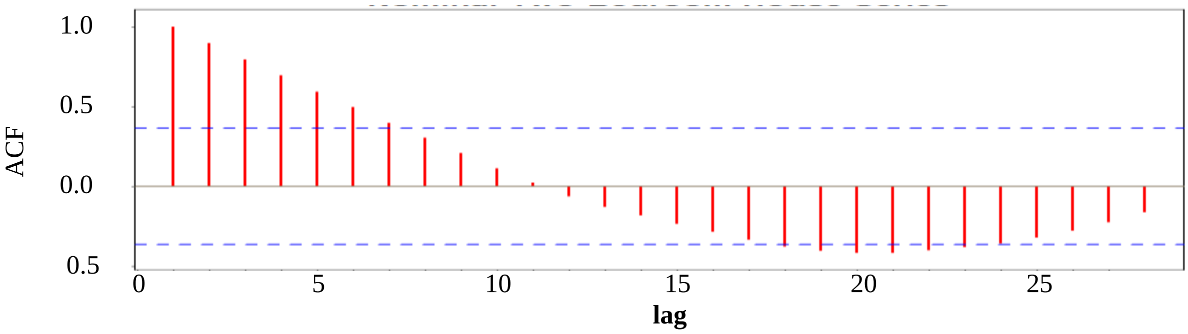

Fig. 2 Non-Stationary ACF Nominal Series for Two-Bedroom House

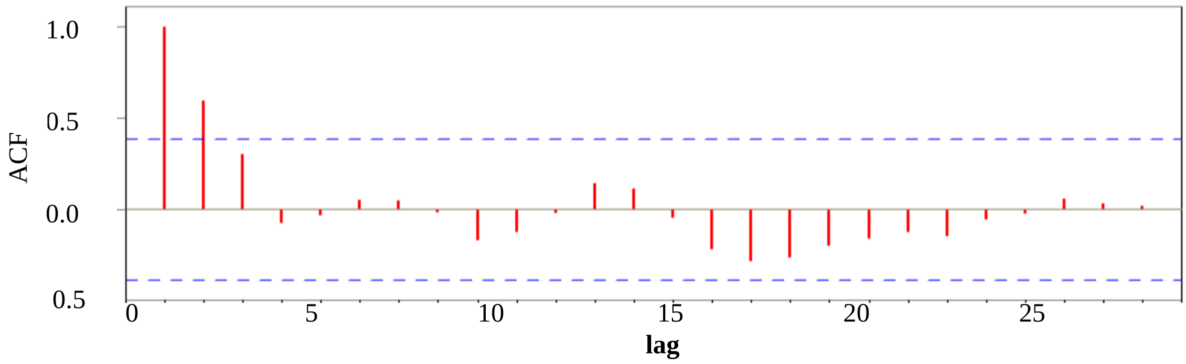

Fig. 3 ACF to Determine Order of Autoregressive Process for One-Bedroom House

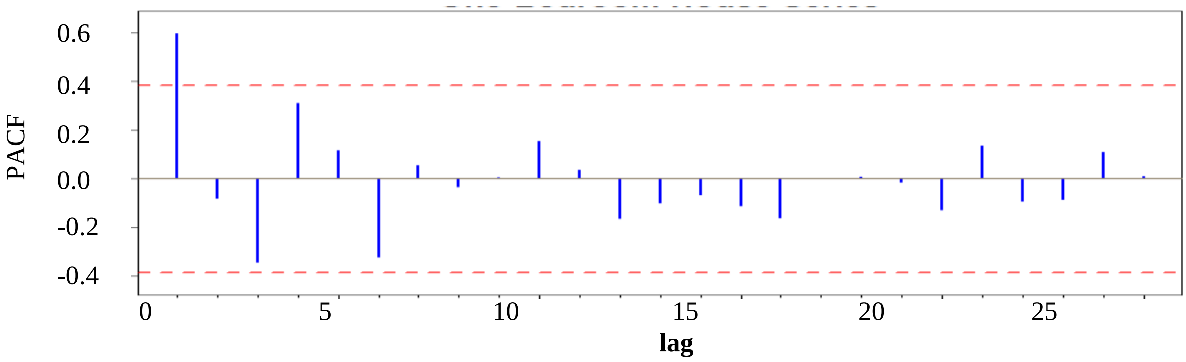

Fig. 4 PACF to Determine Order of Moving Average Process for One-Bedroom House

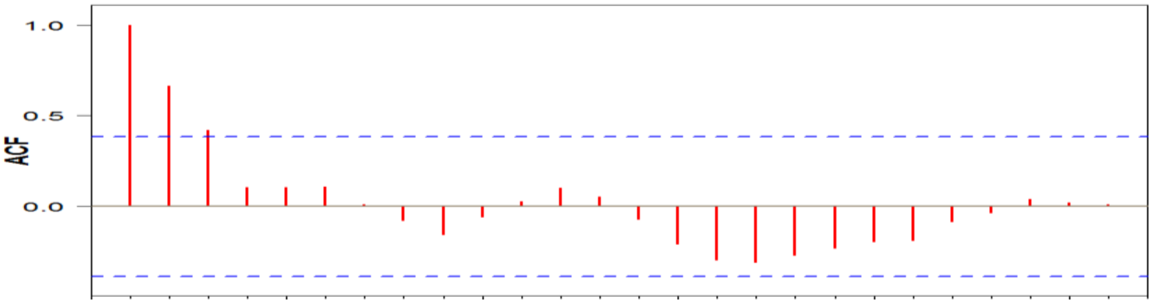

Fig. 5 Using ACF to Determine Order of Autoregressive Process for Two-Bedroom House

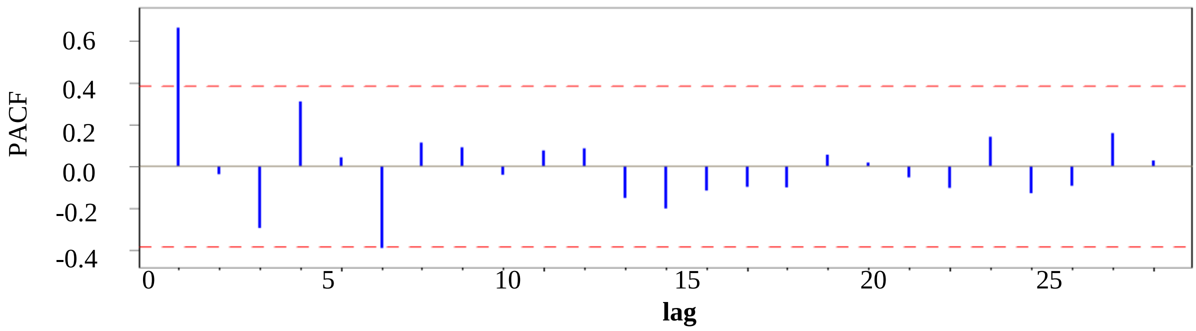

Fig. 6 Using PACF to Determine Order of Moving Average Process for Two-Bedroom House

Table 3 Exchange Rate, Consumer Price Index (CPI) and Nominal Values of Housing Unit Prices

			Nominal Values	
Date	**Exchange Rate**	**CPI**	**One-Bedroom**	**Two-Bedroom**
2003.01	0.8375	28.2400	31 455.00	34 500.00
2003.07	0.8450	34.2300	33 260.00	37 280.00
2004.01	0.8675	36.4200	35 065.00	40 070.00
2004.07	0.9032	39.3600	36 870.00	41 880.00
2005.01	0.9000	42.5400	38 675.00	43 680.00
2005.07	0.9040	46.2000	40 587.50	45 841.00
2006.01	0.9095	47.9600	42 500.00	48 000.00
2006.07	0.9225	52.1700	42 500.00	48 000.00
2007.01	0.9210	53.2000	42 500.00	48 000.00
2007.07	0.9307	57.4600	44 604.00	53 579.00
2008.01	0.9688	60.0100	46 708.00	59 160.00
2008.07	1.1550	67.9900	49 020.00	64 001.00
2009.01	1.3402	71.9100	51 332.00	68 840.00
2009.07	1.4965	81.9000	51 332.00	74 419.00
2010.01	1.4312	82.5500	51 332.00	80 000.00
2010.07	1.4383	89.6400	53 873.00	80 000.00
2011.01	1.5637	90.0600	56 414.00	80 000.00
2011.07	1.5126	97.1800	59 206.50	89 001.00
2012.01	1.6808	97.9000	61 999.00	98 000.00
2012.07	1.9549	106.4500	61 999.00	103 671.00
2013.01	1.9035	106.5000	61 999.00	109 340.00
2013.07	2.0800	113.6000	65 068.00	114 919.00
2014.01	2.3800	121.2000	68 137.00	120 500.00
2014.07	3.4650	131.0000	69 942.00	126 727.00
2015.01	3.3300	141.1000	71 747.50	132 955.00
2015.07	3.8200	154.5000	73 552.00	139 183.00
2016.01	3.9533	168.0000	75 357.50	145 410.00
2016.07	3.8965	180.3000	77 162.00	151 637.00
2017.01	4.3615	190.3400	78 967.50	157 865.00

Source: www.bog.gov.gh, Accessed: July 2, 2017

Table 4 Exchange Rate, Consumer Price Index (CPI) and Real Monetary Values of Housing Unit Prices

			Real Monetary Values	
Date	**Exchange Rate**	**CPI**	**One-Bedroom**	**Two-Bedroom**
2003.01	0.8375	28.2400	26 343.56	28 893.75
2003.07	0.8450	34.2300	23 186.58	25 989.05
2004.01	0.8675	36.4200	23 586.75	26 953.40
2004.07	0.9032	39.3600	23 892.78	27 139.40
2005.01	0.9000	42.5400	23 106.81	26 097.11
2005.07	0.9040	46.2000	22 427.63	25 330.59
2006.01	0.9095	47.9600	22 760.26	25 705.70
2006.07	0.9225	52.1700	21 222.63	23 969.09
2007.01	0.9210	53.2000	20 777.90	23 466.80
2007.07	0.9307	57.4600	20 402.46	24 507.75
2008.01	0.9688	60.0100	21 294.45	26 971.39
2008.07	1.1550	67.9900	23 516.62	30 703.54
2009.01	1.3402	71.9100	27 016.76	36 231.46
2009.07	1.4965	81.9000	26 487.79	38 400.89
2010.01	1.4312	82.5500	25 132.53	39 168.59
2010.07	1.4383	89.6400	24 410.88	36 249.52
2011.01	1.5637	90.0600	27 661.33	39 226.19
2011.07	1.5126	97.1800	26 024.43	39 120.71
2012.01	1.6808	97.9000	30 059.57	47 514.28
2012.07	1.9549	106.4500	32 153.50	53 765.15
2013.01	1.9035	106.5000	31 293.39	55 188.30
2013.07	2.0800	113.6000	33 644.74	59 421.22
2014.01	2.3800	121.2000	37 785.23	66 823.02
2014.07	3.4650	131.0000	52 243.79	94 659.85
2015.01	3.3300	141.1000	47 817.70	88 610.79
2015.07	3.8200	154.5000	51 356.34	97 181.98
2016.01	3.9533	168.0000	50 077.39	96 629.44
2016.07	3.8965	180.3000	47 092.00	92 544.12
2017.01	4.3615	190.3400	51 099.76	102 154.23

Analysis of Model Residuals

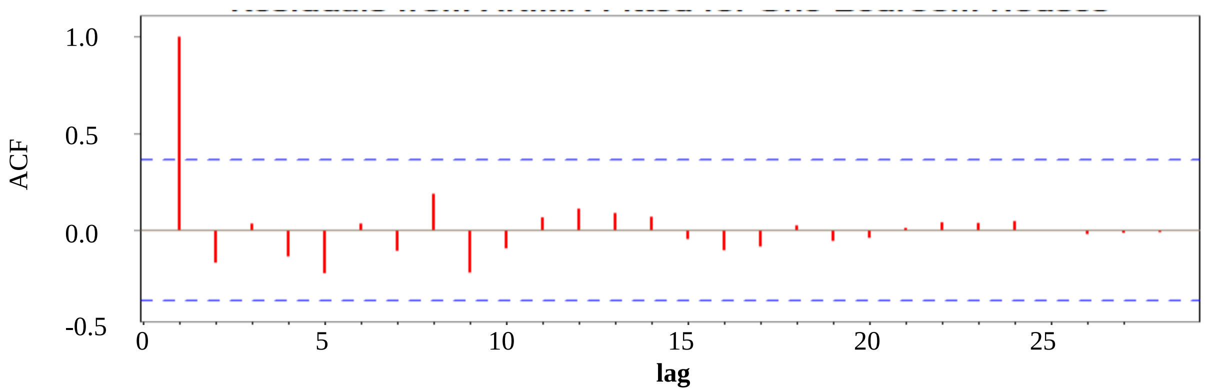

Fig. 7 ACF Residual Plot for ARIMA Model for One-Bedroom House

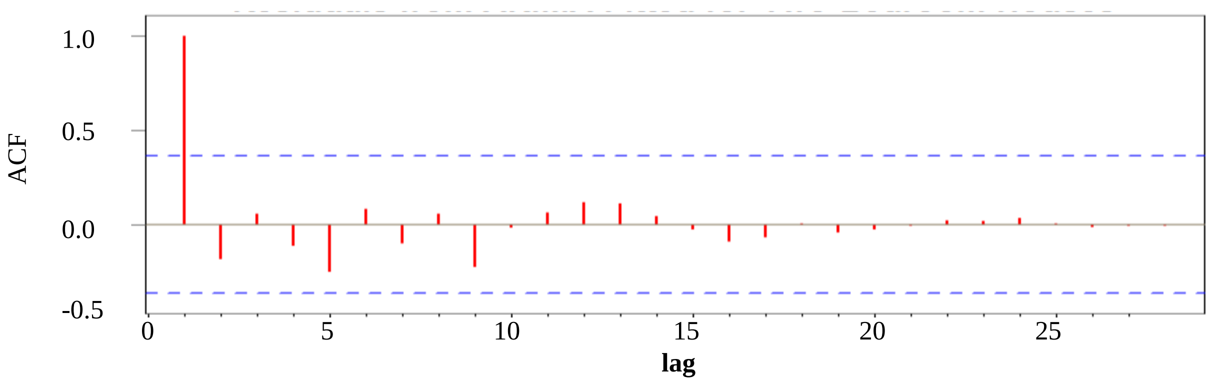

Fig. 8 ACF Residual Plot for ARIMA Model for Two-Bedroom House

Table 5 Augmented Dickey-Fuller Tests for Unit Root for One-Bedroom Housing Unit

Dickey-Fuller	Lag Order	P-Value
-5.0544	2	0.01

Table 6 Augmented Dickey-Fuller Tests for Unit Root for Two-Bedroom Housing Unit

Dickey-Fuller	Lag Order	P-Value
- 4.4982	2	0.01

3 Results and Discussion

The developed Time Series Analysis (TSA) model for one-bedroom and two-bedroom housing units is as shown in Equation (3). After derivation of the models coefficients, θ and ϕ, the respective equations for one-bedroom and two-bedroom housing units are as follows:

$$\hat{HUP}_{t1\text{-}Bed} = 440.531 - 0.181 y_{t-1} + 0.022 y_{t-2} + 0.993\, e_{t-1} \quad (4)$$

and

$$\hat{HUP}_{t2\text{-}Bed} = 278.474 - 0.166\, y_{t-1} + 0.035\, y_{t-2} - 0.062\, y_{t-3} + 0.994\, e_{t-1} \quad (5)$$

The Augmented Dickey-Fuller (ADF) statistic used in the ADF tests are negative numbers, and the more negative they are, the stronger the rejection of the null hypothesis that the series is not stationary at 95% level of confidence. According to Montgomery *et al.* (2008), if a time series computed p-value is less than $\alpha = 0.05$ significance level, the null hypothesis of the series that it is not stationary should be rejected. Tables 5 and 6 show the time series analyses results for the ADF tests as well as their p-values for one-bedroom and two-bedroom housing units respectively. Clearly, it can be seen that the ADF values are negatives and the corresponding p-values are less than $\alpha = 0.05$. This implies that the time series for one-bedroom and two-bedroom housing units are stationary. In Figs. 7 and 8, it can be seen that all the sample autocorrelations except that of the zero lag are within the 95% confidence bounds for the ACF plots of the one-bedroom and two-bedroom housing unit respectively. This means that the residuals are independent identically distributed and they are normally distributed having zero mean and a unit variance. This clearly satisfies the two main important assumptions of the TSA modeling. Consequently, it can be concluded that the developed models fitted the data very well and can be used to predict future housing unit prices for one-bedroom and two-bedroom housing units.

Model Validation

In order to find the efficiency of the developed ARIMA (2, 1, 1) and ARIMA (3, 1, 1) models for one-bedroom and two-bedroom housing units, Equations (4) and (5) were used to estimate the known HUP in the 15.5 year for one-bedroom and two-bedroom housing units. Table 7 is a summary of the results. From the results, the percentage absolute deviations, (Δ %), of the estimates of the HUP from the known HUP are 0.00%, meaning the models are very good.

Table 7 Estimated HUP and Respective Percentage Absolute Deviation (Δ %) from the Known HUP

Housing Unit	Known HUP ($)	Estimated HUP ($) from TSA	Δ %
1-Bedroom	83 600.00	83 618.82	0.00
2-Bedroom	169 000.00	169 104.99	0.00

4 Conclusions and Recommendation

In this paper, TSAM has been developed from observed housing unit prices over a period of 15 consecutive years, obtained from an estate development agency, to determine realistic HUP for one-bedroom and two-bedroom housing units. In developing the TSAM, nonstationarity which existed among the sample data and could have caused wrong statistical inferences was resolved by transforming the nominal housing unit prices to real monetary values of housing unit prices.

The models which determined the HUP for one-bedroom and two-bedroom housing units are

$$\hat{HUP}_{t1\text{-}Bed} = 440.531 - 0.181\, y_{t-1} + 0.022\, y_{t-2} + 0.993\, e_{t-1}$$

$$\hat{HUP}_{t2\text{-}Bed} = 278.474 - 0.166\, y_{t-1} + 0.035\, y_{t-2} - 0.062\, y_{t-3} + 0.994 e_{t-1}$$

respectively.

The percentage absolute deviations, (Δ %), using Equations (4) and (5) to estimate the HUP from the known HUP in the 15.5 year are 0.00% for one-bedroom and two-bedroom housing units respectively, meaning that the developed TSAMs are good.

For future research, it is recommended that the developed TSAM should be improved upon to determine the HUP of other types of housing units since it would give prospective house owners a timely, good idea of the price of a house they intend to purchase.

Acknowledgements

We are thankful to Regimanuel Gray Estate Ltd., an estate development agency in Accra Metropolitan Area for providing the data to make this study a successful one.

References

Boye, P. Mireku-Gyimah, D. and Okpoti, C. A. (2017), "Multiple Linear Regression Model for Estimating the Price of a Housing Unit", *Ghana Mining Journal*, Vol. 17, No. 2, pp. 66 -77.

Boye, P. Mireku-Gyimah, D. and Luguterah, A. (2018), "Principal Components Regression Model for Estimating the Price of a Housing Unit", *Ghana Journal of Technology*, Vol. 3, pp. 17- 23.

Enders, W. (2015), *Applied Econometric Time Series*, 4th edition, Hoboken, NJ: Wiley, pp. 7 - 70.

Gebhard, K., Jürgen, W. and Uwe, H. (2013), *Introduction to Modern Time Series Analysis*, 2nd edition, Springer Heidelberg New York Dordrecht London, pp. 10 - 89.

Montgomery, D. C., Jennings, C. L. and Kulahci, M. (2008), *Introduction to Time Series Analysis and Forecasting*, Hoboken, N.J: Wiley-Interscience (Wiley Series in Probability and Statistics), pp. 73 - 275.

Authors

P. Boye is a Lecturer at the Department of Mathematical Sciences of the University of Mines and Technology (UMaT), Tarkwa, Ghana. He obtained his MSc and BSc degrees from Kwame Nkrumah University of Science and Technology (KNUST), Kumasi, Ghana. He holds a degree of PhD in Mathematics from University of Mines and Technology (UMaT), Tarkwa, Ghana. His research interest is in applied Mathematics and Statistics.

D. Mireku-Gyimah is a Professor of Mining Engineering and a Consulting Engineer currently working at the University of Mines and Technology, Tarkwa, Ghana. He holds the degrees of MSc from the Moscow Mining Institute, Moscow, Russia, and PhD and DIC from the Imperial College of Science, Technology and Medicine, London, UK. He is a member of Institute of Materials, Minerals and Mining of UK and also a fellow of Ghana Institution of Engineers and the Ghana Academy of Arts and Science. His research and consultancy works cover Mine Design and Planning, Mine Feasibility Study, Operations Research, Environmental Protection and Corporate Social Responsibility Management.

H. Sadiq is currently a student at the Department of Statistics of the University of Cape Town, South Africa.