Asset Portfolio Optimisation of Some Selected Equities Using Geometric Mean and Semivariance*

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Abstract

The oldest question on the stock market probably is which portfolio is the best. Fund Managers answer this question using arithmetic mean as a measure of returns on equity and the variance as an appropriate measure of equity risk. However, these two measures have a setback. In this research, we employ the geometric mean and semivariance in an optimal portfolio formation of some selected equities on the Ghana Stock Exchange. A historical data of the best six performing equities in the Ghana Stock exchange in the year 2015 was obtained from the Ghana stock exchange to measure the risk in equity selection. The methods used were geometric mean of the returns on the equity prices, their semivariance, variance, correlation, utility function. Maximisation function and the minimisation function of the semivariance and the sharpe ratio. The results revealed that the equities with the highest Sharpe Ratio were CAL Bank Limited (CAL), Ghana Commercial Bank (GCB), and Enterprise Group Limited (EGL). A minimum variance portfolio of 0.4 GCB, 0.1 CAL, and 0.5 EGL, with portfolio risk of 0.00072 and portfolio returns of 0.02148 with a Sharpe Ratio 0.72758. Efficient frontier portfolio of 0.5 GCB, 0.1 CAL and 0.4 EGL,0.5 GCB, 0.2 CAL and 0.3 EGL,0.5 GCB, 0.3 CAL and 0.2 EGL, 0.5 GCB, 0.4 CAL and 0.1 EGL were obtained.

Keywords: Asset Portfolio, Geometric Mean, Semivariance, Sharpe Ratio

1 Introduction

The fundamental goal of portfolio theory is the optimal allocation of investments between different equities. Mean Variance Optimization (MVO) is a quantitative tool, which enables allocation of investment by considering tradeoff between risk and return.

"The rule that the investor does (or should) maximize discounted expected, or anticipated, returns is rejected both as a hypothesis to explain, and as a maximum to guide investment behavior. We next consider the rule that the investor does (or should) consider expected return a desirable thing and variance of return an undesirable thing. This rule has many sound points, both as a maxim for, and hypothesis about investment behavior.

In our quest to achieve an optimal portfolio, investors may apply two main management approaches, the Passive or Active Portfolio management.

The investor's main purpose in passive portfolio management is to replicate a given financial index as much as possible. The implicit assumption therefore is financial markets are efficient, meaning no financial strategy can regularly outperform their performance. A private investor may choose the Passive approach when the market is presumed to be bullish with relatively high probability. If the financial market is not efficient, a better asset allocation where financial assets are first selected and their weightings optimized, this process is known as active portfolio management. This process includes researching ideas, forecasting exceptional returns, constructing and implementing portfolios, and observing and refining their performance (Prigent, 2007).

Markowitz mean–variance efficiency is the classic paradigm of modern finance for efficiently allocating scarce resources among risky investment instrument. For a given estimates of the expected return, standard deviation or variance, and correlation of return for a set of assets, mean – variance efficiency provides investor with prescription for optimal allocation of resources. The variance or standard deviation defines the portfolio risk whiles the mean is the expected portfolio returns. The mean – variance efficiency is the criterion of choice for defining optimal portfolio structure and for rationalising the value of diversification.

Many investment situations may use mean – variance efficiency for wealth allocation. An international equity manager may want to find optimal asset allocations among international equity markets based on market index historic returns. A plan sponsor may want to find an optimal long-term investment policy for allocating domestic and foreign debt, equities and other asset classes. A domestic equity manager may want to find the optimal equity portfolio based on forecast of return and estimated risk. Mean–variance optimisation is sufficiently flexible to consider various trading cost, institutional and client

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constraints, and desired levels of risk. Mean–variance efficiency serves the standard optimisation framework for modern asset management (Michaud and Michaud, 2008).

Most asset portfolio optimization study employs the arithmetic mean of the returns of an equity as the expected returns of the equity and the variance of the returns as the risk of the equity. Risk – Return Analysis of Optimal Portfolio using Sharpe ratio employs the arithmetic mean of equities as their expected returns and variance as the risk (Boamah, 2012).

The arithmetic mean has dampened effect on high and low values in data set. The variance as a measure of risk is valid only when:

(i) The underlying distribution of the returns is symmetric.

(ii) The underlying distribution of the returns is normal.

Both properties above are seriously questioned by empirical evidence on the subject. Portfolio risk and portfolio returns calculated based on arithmetic mean and variance misrepresent the true return and risk profile of the portfolio. The geometric mean and semivariance are employed to optimise a portfolio of selected equities on the Ghana Stock Exchange to arrive at a true position of risk for any given returns of a portfolio.

The main purpose of this paper was to form a diversified portfolio of selected equities on the Ghana Stock Exchange Mining, construct an optimal portfolio with the three highest Sharpe ratio equities and determine some efficient frontiers for the highest Sharpe ratio equities.

1.1 Literature Review

Modern portfolio theory started with the pioneering work of Harry Markowitz in 1952 which earned him the 1990 Nobel Prize in economics because of the enormous impact of this theory on investment management thereafter.

Tobin (1958) shows that there is a logical connection between the assumption that asset returns are random variables, with variance and expected returns as the main criteria for selecting assets, and the Morgenstern-Von Newman expected utility theory. This implies if an investor behaves as the expected utility theory predicts, then he will choose his portfolio in accordance with the mean variance optimization approach (Markowitz, 1959).

Fletcher and Hillier found a little evidence of higher Sharpe-ratio and abnormal return generated from mean-variance and resampled strategies, (Fletcher and Hillier, 2005).

Lintner (1965) added a justification for the use of the variance in the measurement of risk by presenting a more complete formalization which renders practically the same results as those obtained by Sharpe.

Fama (1970) in line with the Chicago school develops a general framework in order to test Efficient Market Hypothesis which highlights the logical coherence between special cases that are being evaluated.

Hachloufi *et al.* (2012) presented an approach based on the classifications of genetic algorithms for an optimal choice of a reduced size portfolio. This led to a financial gain surplus in terms of cost and taxes reduction, and performance at reduced design loads.

Pandari *et al.* (2012) applied Genetic Algorithm (GA) to select the best portfolio in order to optimize their objectives of the rate of return, return skewness, liquidity and Sharpe ratio. The obtained results were compared with the results of Markowitz classic model. The comparison showed that, the rate of return of the portfolio of GA model was less than that of the Markowitz classic model.

Divya and Kumar (2012) planned to obtain a closer representation for the uncertainty that signalise Financial Market, thereby outlining an approach to solve Financial Assets selection problems for a portfolio in a non-linear and uncertainty environment, by the application of a Fuzzy logic and Genetic Algorithm to optimize the investment profitability.

Sinha *et al.* (2013) generated an algorithm to construct an optimum portfolio from a vast pool of stocks listed in a single market index SPX 500 index. Their algorithm however selects stocks on the grounds of a priority index function created on company fundamentals and genetically give optimum weights to the stocks selected by searching for a genetically appropriate combination of return and risk on the grounds of historical data.

Guha *et al.* (2013) proposed a fuzzy portfolio selection model based on fuzzy linear programming solved by Guhu et al. They provided for finding a global near optimal solution with a reduction in computational complexity compared to the existing methods.

2. Resources and Methods Used

2.1Resources

The data on share prices for five selected equities were collected from the Ghana Stock Exchange for a five-year period from 2010 – 2015. These are monthly data from January, 2010 – December, 2015. The market capitalization is the product of the closing price and shares issued. Table 1 shows the market information of Equities.

Table 1 Market Information of Equities

COMPANY	CLOSING PRICE (31/12/15)	SHARES ISSUED (Mil) - (31/12/15)	MARKET CAPILIZATION (Mil) - (31/12/15)
FML	7.35	116.21	854.14
ETI	0.27	24,067.75	6,498.29
EGL	2.4	133.1	319.44
GCB	3.79	265	1,004.35
GGBL	1.99	211.31	420.51
CAL	1	548.26	548.26

2.2 Methods

2.2.1 Geometric Mean

The geometric mean is the average of a dataset of products, the calculation of which is commonly used to determine the performance results of an investment or portfolio. It is technically defined as the n^{th} root product of 'n' numbers. It must be used when working with percentages, which are derived from values whiles the standard arithmetic mean works with the values themselves.

$$Geometric\ Mean = \left(\prod_{i=1}^{n} x_i\right)^{\frac{1}{n}} = \sqrt[n]{x_1 x_2 \mathrm{K}\ x_3} \qquad (1)$$

2.2.2 Calculation of Risk

The variance of the portfolio simply measures the volatility or fluctuations of returns of the portfolio. It is the measure of surprises the investor is exposed to by holding a portfolio of assets. It is measured by the Variance, the Standard Deviation, the Semivariance and the Semideviation.

Variance

Variance is a measure of the variation of possible rates of return$\left(R_i\right)$, from the expected rate of return $\left(\overline{x}\right)$. This metric measures the volatility of the portfolio. The formula for the computation of an individual investment variance and semivariance is given as Equation (2).

$$\text{Variance } \left(\sigma^2\right) = \sum_{i=1}^{n} \left(x_i - \overline{x}\right)^2 P_i \qquad (2)$$

where P_i is the probability of the possible rate of return x_i.

Semivariance

The semivariance is the measure of the dispersion of all observations that falls below the mean or a target value of a data set. Semivariance is an average of the squared deviations of values that are less than the mean. Semivariance is similar to variance, however it only considers observations below the average or a target value (Huang, 2008). It is a useful tool in portfolio or asset analysis, semivariance provides a measure for downside risk while standard deviation and variance provide measures of volatility. Semivariance only looks at the negative fluctuations of an asset. By neutralizing all values above the mean or an investor's target return, semivariance estimates the average losses the portfolio could incur. For risk, adverse investors, solving for optimal portfolio by minimising semivariance would limit the likelihood of a large loss (Sharpe, 1963).
The semivariance formula is given by

$$\text{Semivariance} = \frac{1}{n} \sum_{x_i < \overline{x}}^{n} \left(\overline{x} - x_i\right)^2 \qquad (3)$$

2.2.3 Sharpe Ratio

The Sharpe ratio is a measure of stock of fund performance, it measures the reward per unit of risk. By definition it is the ratio of an asset's excess return to its volatility. It is also known as the reward-to-variability ratio. The Sharpe ratio computed based on realized returns is as follows

$$S_j = \frac{\hat{\mu}_j - r_j}{\partial_j} \qquad (4)$$

where $\hat{\mu}_j$ is the return on asset j, r_f is the return on risk free asset, $\hat{\sigma}_j$ is the semi deviation of $\hat{\mu}_j$.
The portfolio that maximizes the Sharpe ratio is known as The Sharpe optimal portfolio and is given as:

$$S^* = \arg\max \frac{\hat{\mu}_j - r_j}{\partial_j} \quad where \left(J : \sum_{i=1}^{d} j_i = 1\right) \ (5)$$

2.2.4 Assets and Portfolios

Anything we can purchase can be termed asset. An asset is an economic resource that can be owned and is expected to provide future economic benefits.

Asset prices sometimes seem to deviate from what fundamentals would suggest and exhibit patterns different than predictions of standard models with perfect financial markets. A bubble, an extreme form of such deviation, can be defined as the part of a grossly upward asset price movement that is unexplainable based on fundamentals (Garber, 2000).

Let $P(t)$ denote Random price at time t, $R(t)$ denote Random gross return at time t and $r(t)$ denote Random net return at time t.

$$R(t)=\frac{P(\mathrm{t}+1)}{P(t)} \tag{6}$$

where $P(t+1)$ denotes the random price at time $t+1$

$$r(t)=R(t)-1=\frac{P(\mathrm{t}+1)-\mathrm{P}(t)}{P(t)} \tag{7}$$

Let W denote the total wealth distributed over d assets; $W>0$ $w_i=$dollar amount in asset i: $w_i>0\equiv\sim long, w_i<0\equiv short$

The net return on a position w is given by

$$r_w(t)=\frac{\sum_{i=1}^{d}R_i(t)w_i-\sum_{i=1}^{n}w_i}{W}=\frac{\sum_{i=1}^{d}r_i(t)w_i}{\sum_{i}^{d}w_i}=\sum_{i=1}^{d}r_i(t)\times\frac{w_i}{W} \tag{8}$$

where $\frac{w_i}{W}=x_i$ and

$$r_x=\sum_{i=1}^{d}r_i x_i \tag{9}$$

Let $x=(x_1,x_2,\ldots,x_d)$:each component can be either positive or negative. x_i is the fraction invested in asset $i\rightarrow\sum_{i=1}^{d}x_i$ .The random net return on the portfolio is given by $r_x=\sum_{i=1}^{d}r_i x_i$

Portfolio vector

2.2.5 Reduction of Uncertainties of Diversification

Let d be total assets with

$\mu_i\equiv\mu,\sigma_{ij}\equiv\sigma,\rho_{ij}=0$ *for all* $i\neq j$

Given two portfolios of asset $x=(1,0,\ldots,0)^T$

Everything invested in asset 1

$y=\frac{1}{d}(1,1,\ldots,1)^T$:Equal all investment in all assets.

Expected returns of portfolios x and y

$$\mu_y=E\left[\sum_{i=1}^{d}\mu_i x_i\right]=\mu_i=\mu \tag{10}$$

$$\mu_y=E\left[\sum_{i=1}^{d}\mu_i y_i\right]=\frac{1}{d}\sum_{i=1}^{d}\mu_i=\mu \tag{11}$$

Both portfolios have the same returns

Semivariance of returns of both portfolios are given by

$$\sigma_x^2=\mathrm{var}\left(\sum_{i=1}^{d}r_i x_i\right)=\sigma^2 \tag{12}$$

$$\sigma_x^2=\mathrm{var}\left(\sum_{i=1}^{d}r_i y_i\right)=\sum_{i=1}^{d}\sigma^2\left(\frac{1}{d}\right)^2=\frac{\sigma^2}{d} \tag{13}$$

2.2.6 Capital Asset Pricing Model (CAPM) as Pricing Formula

Suppose the payoff from an investment in 1yr is X , what is the fair price of this investment?

Let $r_X=\frac{X}{p}-1$ denote the net rate of return on X.

The Beta or the market risk of X is given by

$$\beta_X=\frac{\mathrm{cov}(r_X,r_m)}{\sigma_m^2}=\frac{1}{p}\frac{\mathrm{cov}(X,r_m)}{\sigma_m^2}$$

Suppose CAPM holds, then $\mu_X=E[r_x]$ must lie on the security market line implying

$$\mu_X=r_f+\beta_x(r_m-r_f) \tag{14}$$

$$\frac{E[X]}{P}-1=r_f+\frac{1}{P}\frac{Cov(X,r_m)}{Var(r_m)}(\mu_m-r_f) \tag{15}$$

$$P=\frac{E(X)}{rf+1}-\frac{Cov(X,r_m)}{(1+rf)Var(r_m)}(\mu_m-r) \tag{16}$$

2.2.7 Portfolio Optimisation

There are two obvious formulations for the portfolio optimization.

Maximization of geometric returns:

$Max\{w^T\mu\}$

Subject to
$$w^T\sum w\leq\alpha$$
$$I^T w=1$$

Minimisation of risk:

$$Min\left\{w^T \sum w\right\}$$

Subject to

$$w^T \mu \geq \beta$$
$$I^T w = 1$$

2.2.8 Trend Analysis

Trend analysis is the process of comparing data over time to identify any consistent results or trends. It is also a statistical technique that uses historical results to predict future outcomes. It is based on the idea that what has happened in the past gives an idea of what is likely to happen in the future.

Trend Equation

A simple trend line equation is given by:

$$y = \beta_0 + \beta_1 t \tag{17}$$

where β_0 is the y-intercept and β_1 is the slope with time.

3 Results and Discussion

3.1 Descriptive Statistics

Table 2 shows the kurtosis of the selected equities. The standard reference for kurtosis in return distribution is the normal distribution, which has a kurtosis coefficient of 3. The higher the kurtosis coefficient, the higher the level of kurtosis, so a kurtosis coefficient of 4 would indicate a relatively peaked return distribution, while a kurtosis coefficient of 2 would indicate a relatively flat return distribution. Except ETI and CAL, all other equities recorded kurtosis of 3 and above, which indicate that the returns do not follow the normal distribution.

Table 2 Kurtosis and P-Values Equities on the Ghana Stock Exchange

Equity	Kurtosis On Returns	Geometric Mean on Returns	P-Value
FML	21.18	0.003	0.03
ETI	2.63	0.008	0.03
EGL	5.98	0.019	0.03
GCB	5.29	0.023	0.03
GGBL	3.84	0.006	0.02
CAL	1.25	0.022	0.03

3.1.1 Trend Analysis

The volatility result from the price hikes over period hence, it is intense, and large fluctuations are followed by large fluctuations indicating variation in variance. Fig. 1 shows the returns volatility of the selected equities.

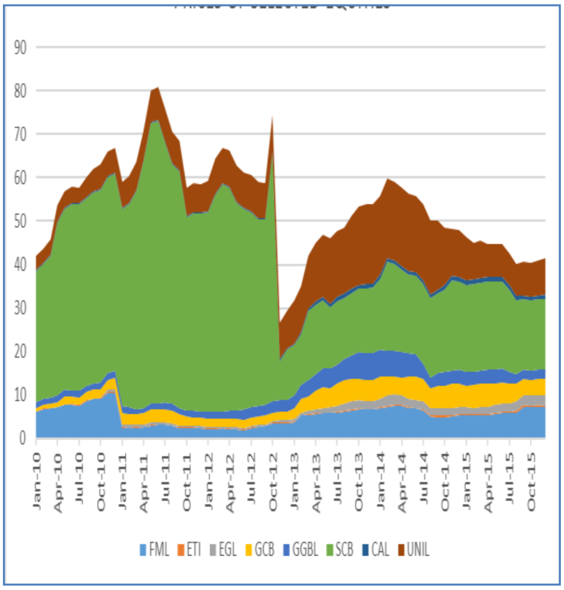

Fig. 1 Stocked Area Chart of Equity Prices of Selected Equities on the Ghana Stock Exchange

3.2 Returns and Risk on Equities

ETI, EGL, GCB and CAL had their returns exceeding their risk. GGBL exposes an investor to similar risk and returns whiles an investor investing in FML is extremely exposed. However, the higher the exposure, the higher the gambled reward. Table 3 and Fig. 2 shows detailed information on returns against risk of the equities.

Table 3 Geometric Mean and Semi Variance of some Selected Equities on the Ghana Stock Exchange

SHARE CODE	GEOMETRIC MEAN	SEMI VARIANCE
FML	0.8254	0.0161
ETI	0.5602	0.0044
EGL	0.2465	0.0028
GCB	0.1971	0.0029
GGBL	0.6771	0.0057
CAL	0.2046	0.0026
GSE-ASI	2.7791	0.0258

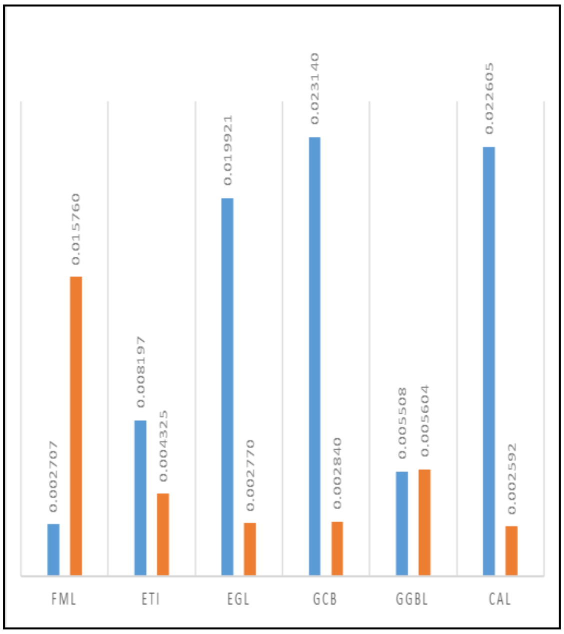

Fig. 2 Bar graph of Returns and Risk

3.3 Correlation Coefficient of Selected Equities

The only combination with a weak negative correlation implying they are moving in opposite direction but not strong enough is FML and CAL. All other combinations are moving in the same direction hence the positive correlation. The highest correlation recorded was GCB and CAL, this combination recorded a correlation of 0.469895. This might have resulted from the fact that they are both in the banking industry. Table 4 Shows the correlation between equities.

3.4 Sharpe Ratio

CAL recorded the highest Sharpe ratio of 0.41034 followed by GCB and EGL respectively. The Sharpe ratio simply indicate the excess returns of a given equity over the risk-free instrument. CAL, GCB and EGL outperform the risk-free instrument (182 Treasury Bill) more compared to the performance of FML, ETI and GGCB over the risk-free instrument. Table 5 shows some selected equities and their corresponding Sharpe Ratio, and Fig. 3 shows the bar chart of the equities and their corresponding Sharpe Ratio.

Table 5 Selected Equities and Their Corresponding Sharpe Ratio

Share Code	Sharpe Ratio
FML	0.00623
ETI	0.09635
EGL	0.34441
GCB	0.40137
GGBL	0.04806
CAL	0.41034

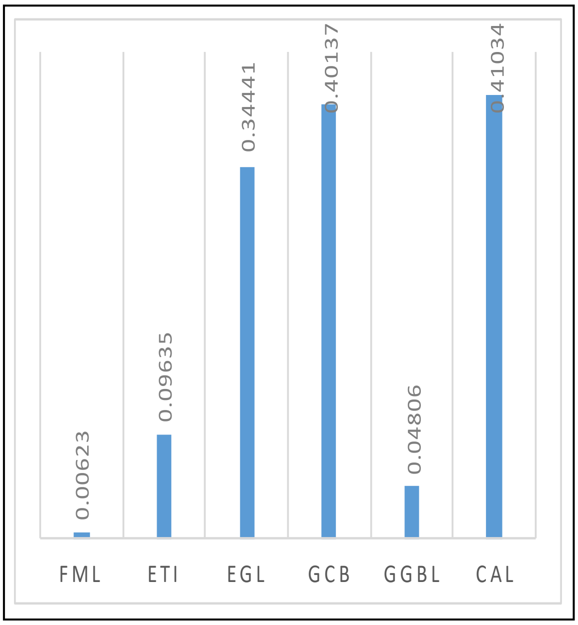

Fig. 3 Bar Graph of Sharpe Ratio of Stocks

Table 4 Correlation Coefficient of Selected Equities

	FML	ETI	EGL	GCB	*GGBL*	*CAL*
FML	1	0.069789	0.240202	0.217505	0.183307	-0.18235
ETI	0.069789	1	0.080451	0.213346	0.087917	0.175465
EGL	0.240202	0.080451	1	0.004018	0.109313	0.153677
GCB	0.217505	0.213346	0.004018	1	0.33306	0.469895
GGBL	0.183307	0.087917	0.109313	0.33306	1	0.376415
CAL	-0.18235	0.175465	0.153677	0.469895	0.376415	1

3.5 Portfolio Optimisation

The portfolio optimisation for FML and ETI. The minimum variance portfolio for FML and ETI is 100% ETI and 0% FML with a risk of 0.00271 and returns of 0.01576. the efficient frontier portfolio is 0.1 FML plus 0.9 ETI, 0.2 FML plus 0.8 ETI. For a risk lover, any of the efficient frontiers would be welcomed. Table 6 shows the detailed feasible combination of FML and ETI. Fig. 4 shows the minimum variance portfolio and efficient frontiers for FML and ETI.

Table 6 Geometric Mean – Semi-Deviation of FML and ETI

FML	ETI	Mean	Variance	Stdev
0%	100.00%	0.00820	0.00433	6.58%
10%	90.00%	0.00765	0.00385	6.20%
20%	80.00%	0.00710	0.00373	6.11%
30%	70.00%	0.00655	0.00398	6.31%
40%	60.00%	0.00600	0.00458	6.77%
50%	50.00%	0.00545	0.00555	7.45%
60%	40.00%	0.00490	0.00687	8.29%
70%	30.00%	0.00435	0.00855	9.25%
80%	20.00%	0.00381	0.01060	10.29%
90%	10.00%	0.00326	0.01300	11.40%
100%	0.00%	0.00271	0.01576	12.55%

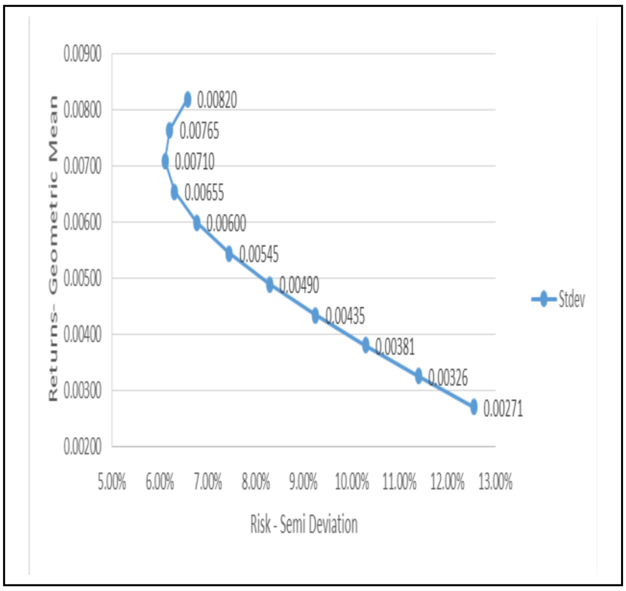

Fig. 4 Efficient Frontier for FML and ETI

3.5.2 Portfolio optimization for FML and EGL

The minimum variance portfolio for FML and EGL is 100% EGL and 0% FML with a risk of 0.00277 and returns of 0.01992. the efficient frontier portfolio is 0.1 FML plus 0.9 EGL, 0.2 FML plus 0.8 EGL. For the risk lover, any of the efficient frontier would be welcomed. Table 7 shows the detailed feasible combination of FML and EGL. Fig. 5 shows the minimum variance portfolio and efficient frontiers for FML and EGL.

Table 7 Portfolio Returns and Deviation FML and EGL

FML	EGL	Mean	Variance	St. dev
0%	100.00%	0.01992	0.00277	5.26%
10%	90.00%	0.01820	0.00304	5.51%
20%	80.00%	0.01648	0.00354	5.95%
30%	70.00%	0.01476	0.00426	6.53%
40%	60.00%	0.01304	0.00522	7.22%
50%	50.00%	0.01131	0.00640	8.00%
60%	40.00%	0.00959	0.00782	8.84%
70%	30.00%	0.00787	0.00946	9.73%
80%	20.00%	0.00615	0.01133	10.64%
90%	10.00%	0.00443	0.01343	11.59%
100%	0.00%	0.00271	0.01576	12.55%

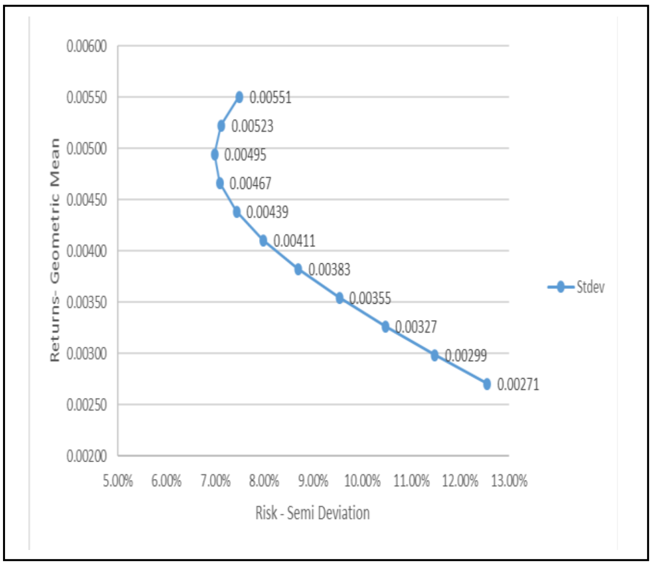

Fig. 5 Efficient Frontier-FML and EGL

3.5.3 Portfolio Returns - FML and GCB

The minimum variance portfolio for FML and GCB is 100% GCB and 0% FML with a risk of 0.00284 and returns of 0.02314. the efficient frontier portfolio is 0.1 FML plus 0.9 GCB, 0.2 FML plus 0.8 GCB. For the risk lover, any of the efficient frontier would be welcomed. Table 8 shows the detailed feasible combination of FML and GCB. Fig. 6 Shows the minimum variance portfolio and efficient frontiers for FML and GCB.

Table 8 Portfolio Returns and Deviations - FML and GCB

FML	GCB	Mean	Variance	St. dev
0%	100.00%	0.02314	0.00284	5.33%
10%	90.00%	0.02110	0.00304	5.51%
20%	80.00%	0.01905	0.00348	5.90%
30%	70.00%	0.01701	0.00417	6.46%
40%	60.00%	0.01497	0.00510	7.14%
50%	50.00%	0.01292	0.00627	7.92%
60%	40.00%	0.01088	0.00768	8.76%
70%	30.00%	0.00884	0.00934	9.66%
80%	20.00%	0.00679	0.01123	10.60%
90%	10.00%	0.00475	0.01338	11.57%
100%	0.00%	0.00271	0.01576	12.55%

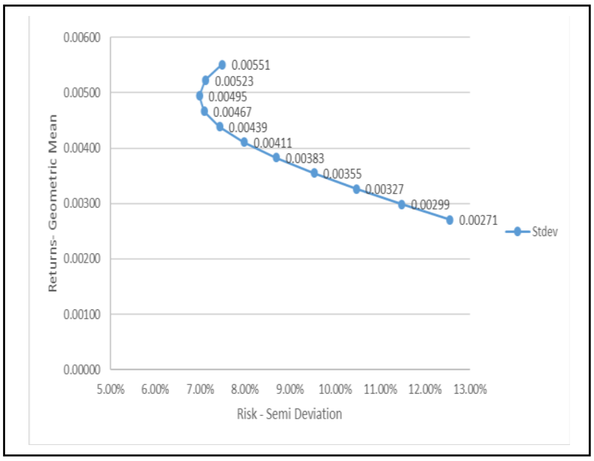

Fig. 6 Plot of Efficient Frontier - FML and GBC

3.5.4 Portfolio Returns and Deviation - FML and GGBL

The minimum variance portfolio for FML and GGBL is 100% GGBL and 0% FML with a risk of 0.0028 and returns of 0.0234. the efficient frontier portfolio is 0.1 FML plus 0.9 GGBL, 0.2 FML plus 0.8 GGBL. For the risk lover, any of the efficient frontier would be welcomed. Table 9 shows the detailed feasible combination of FML and GGBL. Fig. 7 shows the minimum variance portfolio and efficient frontiers for FML and GGBL.

Table 9 Portfolio Returns and Deviation - FML and GGBL

FML	GGBL	Mean	Variance	St. dev
0%	100.00%	0.00551	0.00560	7.49%
10%	90.00%	0.00523	0.00506	7.12%
20%	80.00%	0.00495	0.00487	6.98%
30%	70.00%	0.00467	0.00502	7.09%
40%	60.00%	0.00439	0.00552	7.43%
50%	50.00%	0.00411	0.00636	7.98%
60%	40.00%	0.00383	0.00755	8.69%
70%	30.00%	0.00355	0.00908	9.53%
80%	20.00%	0.00327	0.01096	10.47%
90%	10.00%	0.00299	0.01319	11.48%
100%	0.00%	0.00271	0.01576	12.55%

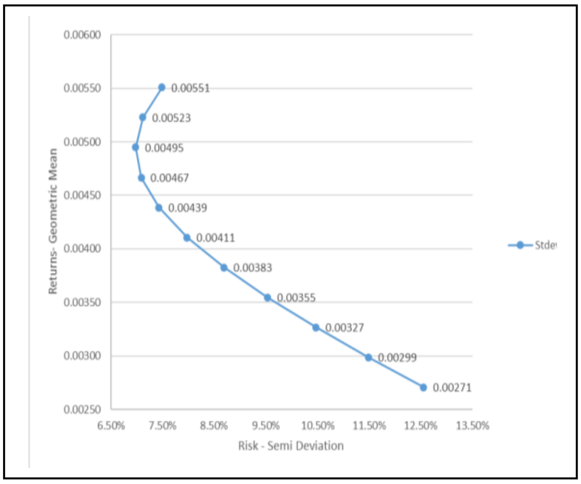

Fig. 7 Efficient Frontier – FML and GGBL

3.5.5 Portfolio Returns and Deviation - FML and CAL

The minimum variance portfolio for FML and CAL is 100% CAL and 0% FML with a risk of 0.00271 and returns of 0.01576. the efficient frontier portfolio is 0.1 FML plus 0.9 CAL, 0.2 FML plus 0.8 CAL. For the risk lover, any of the efficient frontier would be welcomed. Table 10 shows the detailed feasible combination of FML and CAL. Fig. 8 Shows the minimum variance portfolio and efficient frontiers for FML and CAL.

Table 10 Portfolio Returns and Deviation - FML and CAL

FML	CAL	Mean	Variance	St. dev
0%	100.00%	0.204548	0.002613252	5.11%
10%	90.00%	0.184369	0.002044851	4.52%
20%	80.00%	0.16419	0.001902969	4.36%
30%	70.00%	0.144011	0.002187604	4.68%
40%	60.00%	0.123831	0.002898758	5.38%
50%	50.00%	0.103652	0.004036429	6.35%
60%	40.00%	0.083473	0.005600619	7.48%
70%	30.00%	0.063294	0.007591327	8.71%
80%	20.00%	0.043114	0.010008553	10.00%
90%	10.00%	0.022935	0.012852297	11.34%
100%	0.00%	0.002756	0.016122559	12.70%

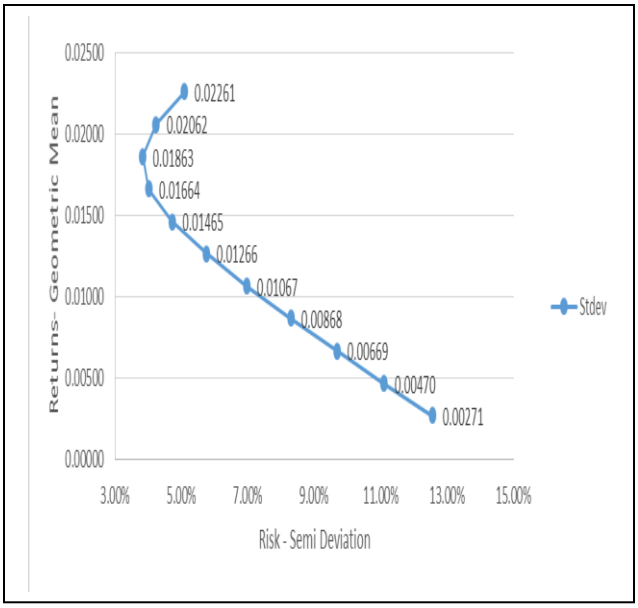

Fig. 8 Efficient Frontier – FML and CAL

3.5.5 Portfolio Returns and Deviation GCB CAL EGL

The minimum variance portfolio for GCB, CAL and EGL is 40% GCB ,10% CAL and 50% EGL, with a risk of 0.00072 and returns of 0.02148 the efficient frontier portfolios are 0.5 GCB plus 0.1 CAL plus 0.4 EGL, 0.5 GCB plus 0.2 CAL plus 0.3 EGL, 0.5GCB plus 0.3 CAL and 0.2 EGL, 0.5GCB plus 0.4 CAL plus 0.1EGL. For the risk lover, any of the efficient frontier would be welcomed. Table 11 shows the detailed feasible combination of GCB, CAL and EGL. Fig. 9 shows the minimum variance portfolio and efficient frontiers for GCB, CAL and EGL.

Table 11 Portfolio Returns and Deviation

GCB	CAL	EGL	Portfolio Returns	Portfolio Risk
40%	10.00%	50.00%	0.02148	0.00072
50%	10.00%	40.00%	0.02180	0.00080
30%	20.00%	50.00%	0.02142	0.00117
10%	40.00%	50.00%	0.02132	0.00132
20%	30.00%	50.00%	0.02137	0.00137
10%	50.00%	40.00%	0.02158	0.00145
50%	20.00%	30.00%	0.02207	0.00151
20%	50.00%	30.00%	0.02191	0.00187
50%	30.00%	20.00%	0.02234	0.00213
30%	50.00%	20.00%	0.02223	0.00229
50%	40.00%	10.00%	0.02260	0.00267
40%	50.00%	10.00%	0.02255	0.00271

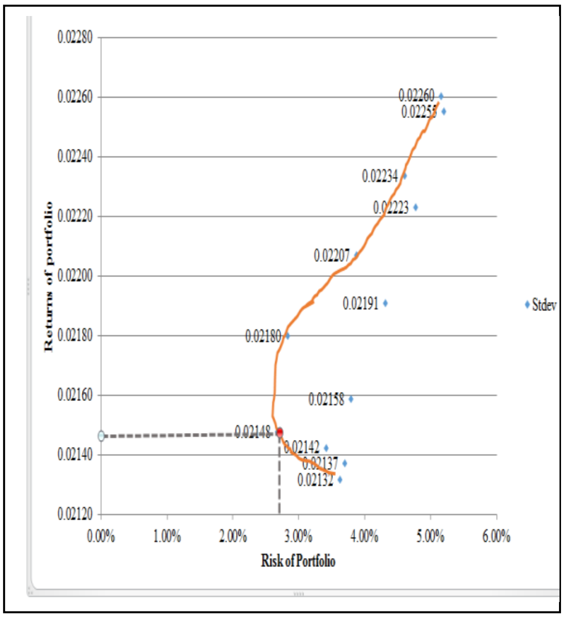

Fig. 9 Efficient Frontier – CAL and GCB and EGL

4 Conclusions and Recommendations

It can be concluded that a diversified portfolio of selected equities on the Ghana Stock Exchange has been formed. The optimal portfolio is given by

$$Min.Var.\ Portfolio = 0.4GCB + 0.1CAL + 0.5EGL \quad (18)$$

(i) with 2% returns at 0.001 risk level.

(ii) The risk level for a given level of returns of minimum variance portfolio has been reduced to 0.001 which is 0.002 less than risk level of any individual equity under study in this research.

(iii) the minimum variance portfolio and efficient frontiers for the three maximum Sharpe ratio equities has been determined as shown in Table 12.

It is recommended that, the best equities obtained in this study should be used for personal and corporate investment portfolio decisions.

Also, citizens should be educated on the risk involved in choosing an equity to invest in.

Table 12 Minimum Variance Portfolio and Efficient Frontiers for the Three Maximum Sharpe Ratio Equities

Portfolio weight	Return	Risk	Sharpe Ratio
0.4GCB+0.1CAL +0.5EGL	0.02148	0.00072	0.7275
0.5GCB+0.1CAL +0.4EGL	0.02180	0.00080	0.6997
0.5GCB+0.2CAL +0.3EGL	0.02207	0.00151	0.5181
0.5GCB+0.3CAL +0.2EGL	0.02234	0.00213	0.4418
0.5GCB+0.4CAL +0.1EGL	0.02260	0.00267	0.3997

References

Boamah, E. K., Kotova, E., Garabedian, M., Jarnik, M. and Tulin, A.V. (2012), *Poly Polymerase 1 (PARP-1) Regulates Ribosomal Biogenesis in Drosophila* nucleoli. *PLoS genetics*, Vol. 8, No. 1, pp. 100- 442.

Divya, P. and Kumar, P. R. (2012), "The Investment Portfolio Selection using Fuzzy Logic and Genetic Algorithm", *International Journal of Engineering Research and Application*, Vol. 2, No. 5, pp. 2100-2105.

Fama, E. (1970), "Efficient Capital Markets: A Review of Theory and Empirical Work", *Journal of Finance*, pp. 383-417.

Fletcher, J. and Hillier, D. J., (2005), "An examination of resampled portfolio efficiency", *Financial Analyst Journal*, pp. 66-89.

Garber, P. M., 2000, *Famous First Bubbles: The Fundamentals of Early Manias*, Cambridge, MA: MIT Press, 117 pp.

Guha, R. and Alier, J. M. (2013), *Varieties of Environmentalism: Essays North and South*. Routledge, 305 pp.

Hachloufi, M. El., Guennoun, Z. and Hamza, F. (2012), "Stocks Portfolio Optimization using Classification and Genetic Algorithms", *Applied Mathematical Sciences*, Vol. 6, No. 94, pp. 4673-4684.

Huang, X. (2008), "Mean-Semivariance Model for Fuzzy Portfolio Selection", *Journal of Computational and Applied Mathematics*, Vol. 217, No. 1, pp. 1-8.

Lintner, J. (1965), "The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets", *The Review of Economics and Statistics*, Vol. 45, No. 1, pp. 13-37.

Lintner, J. (1965), "Security Prices, Risk, and Maximal Gains from Diversification", *The Journal of Finance*, pp. 587-615.

Markowitz, H., (1952), "Portfolio Selection", *The Journal of Finance*, Vol. 7, No. 1, pp. 77-91.

Markowitz, H. M., (1959), *Porfolio Selection: Efficient Diversification of Investments.* New York: John Wiley and Sons, pp. 40-90.

Michaud, R. O. and Michaud, R. O. (2008), *Efficient Asset Management: A Practical Guide to Stock Portfolio Optimization and Asset Allocation*, Oxford University Press, 304 pp.

Prigent, J. L. (2007), *Portfolio Optimization and Performance Analysis*, CRC Press, 301pp.

Pandari, A. R., Azar, A. and Shavazi, A. R. (2012), "Genetic Algorithms for Portfolio Selection Problems with Non-Linear Objectives", *African Journal of Business Management*, Vol. 6, No. 206, 620 pp.

Sharpe, W., (1963), "A Simplified Model for Portfolio Analysis", *Management Science*, Vol. 79, pp. 277-231.

Tobin, J., (1958), "Liquidity Preference as Behavior towards Risk", *A Review of Economics Studies*,Vol 25, No. 2, pp. 65-86.

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